Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER					

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **October/November 2021**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O on a smooth horizontal plane. A particle P of mass m is attached to the other end of the string and moves in a horizontal circle with centre O. The speed of P is $\sqrt{\frac{4}{3}ga}$.

Find the extension of the string. [4]

2 A particle P of mass m kg moves along a horizontal straight line with acceleration $a\,\mathrm{m\,s^{-2}}$ given by

$$a = \frac{v(1-2t^2)}{t},$$

where $v\,\mathrm{m\,s^{-1}}$ is the velocity of P at time t s.

(a) Find an expression for v in terms of t and an arbitrary constant. [3]

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(b) Given that $a = 5$ when $t = 1$, find an expression, in terms of m and t, for the horizontal force acting on P at time t. [3]

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3 A light elastic string has natural length a and modulus of elasticity $12mg$. One end of the string is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle hangs in equilibrium vertically below O. The particle is pulled vertically down and released from rest with the extension of the string equal to e, where $e > \frac{1}{3}a$. In the subsequent motion the particle has speed $\sqrt{2ga}$ when it has ascended a distance $\frac{1}{3}a$.

Find e in terms of a. [6]

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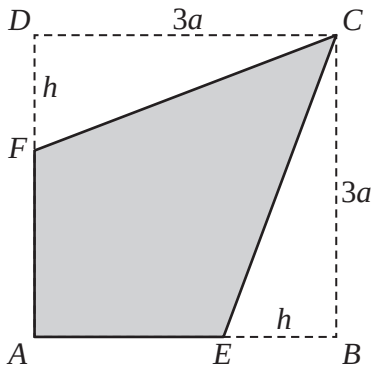

A uniform lamina *AECF* is formed by removing two identical triangles *BCE* and *CDF* from a square lamina *ABCD*. The square has side $3a$ and $EB = DF = h$ (see diagram).

(a) Find the distance of the centre of mass of the lamina *AECF* from *AD* and from *AB*, giving your answers in terms of a and h. [5]

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The lamina *AECF* is placed vertically on its edge *AE* on a horizontal plane.

(b) Find, in terms of a, the set of values of h for which the lamina remains in equilibrium. [3]

5 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. Its initial speed is $u\,\mathrm{m\,s^{-1}}$ and its angle of projection is $\sin^{-1}(\frac{4}{5})$ above the horizontal. At time 8 s after projection, P is at the point A. At time 32 s after projection, P is at the point B. The direction of motion of P at B is perpendicular to its direction of motion at A.

Find the value of u. [7]

6 A particle P, of mass m, is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P moves in complete vertical circles about O with the string taut. The points A and B are on the path of P with AB a diameter of the circle. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O. The speed of P when it is at A is $\sqrt{5ag}$.

The ratio of the tension in the string when P is at A to the tension in the string when P is at B is 9 : 5.

(a) Find the value of $\cos\theta$. [6]

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(b) Find, in terms of a and g, the greatest speed of P during its motion. [2]

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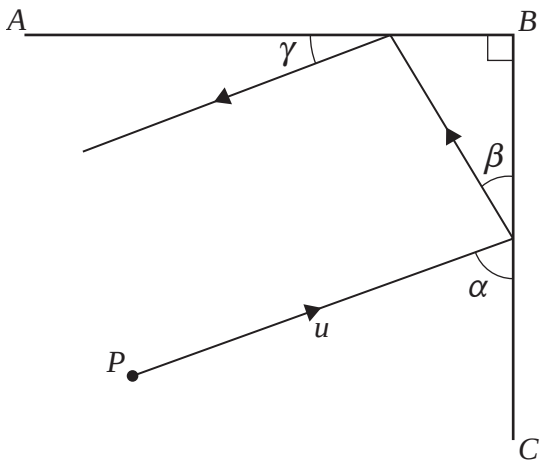

The smooth vertical walls AB and CB are at right angles to each other. A particle P is moving with speed u on a smooth horizontal floor and strikes the wall CB at an angle α. It rebounds at an angle β to the wall CB. The particle then strikes the wall AB and rebounds at an angle γ to that wall (see diagram). The coefficient of restitution between each wall and P is e.

(a) Show that $\tan\beta = e\tan\alpha$. [3]

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(b) Express γ in terms of α and explain what this result means about the final direction of motion of P. [4]

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As a result of the two impacts the particle loses $\frac{8}{9}$ of its initial kinetic energy.

(c) Given that $\alpha + \beta = 90°$, find the value of e and the value of $\tan\alpha$. [4]

Additional Page

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